XXXVII SUMMER SCHOOL ON MATHEMATICAL PHYSICS, RAVELLO, ITALY, SEPTEMBER 17 -29, 2012

#6
Applications.
Available software.

Igor Podlubny
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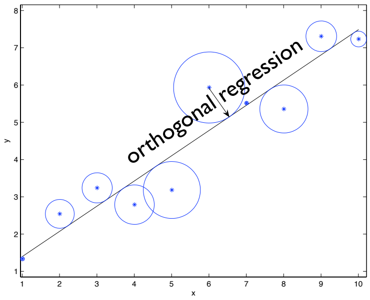
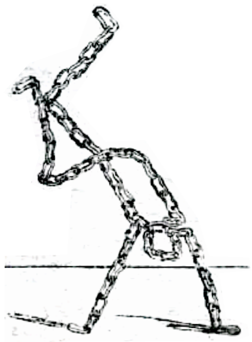
http://www.tuke.sk/podlubny

The Method of Least Circles!

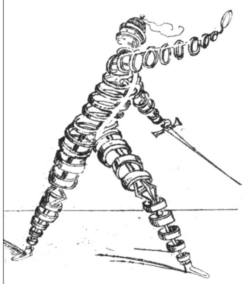
minimize squares of distances

$$E = \sum_i \pi\,[d((x_i, y_i),\, f(x, \alpha_1, \alpha_2, \ldots, \alpha_n))]^2$$

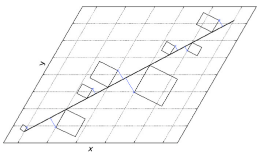




Arguments for orthogonal regression

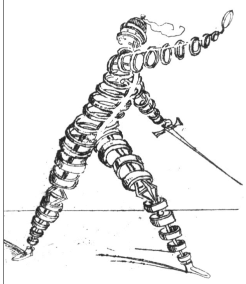


1. The shortest (orthogonal) distance is the most natural viewpoint on any fitting.
2. The sum of orthogonal distances is invariant with respect to the choice of the system of coordinates.

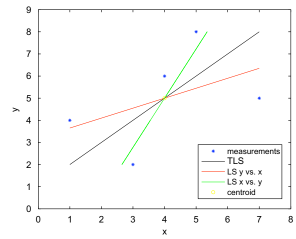


The squares in non-Cartesian coordinates have even less meaning.

Arguments for orthogonal regression



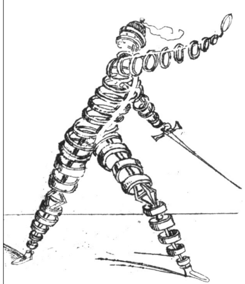
3. There are no conjugate regression lines, which appear after swapping x and y, because in the case of orthogonal regression the fitting $y = f(x)$ gives exactly the same line as the fitting $x = f^{-1}(y)$.



Nievergelt's example (1994)

x	1	3	4	5	7
y	4	2	6	8	5

Arguments for orthogonal regression



4. There are no problems with causality (normally, determination of what is an independent variable and what is a dependent variable is simply unclear or even impossible; this is always postulated).

5. Implementation of the orthogonal fitting does not depend on the number of dimensions.

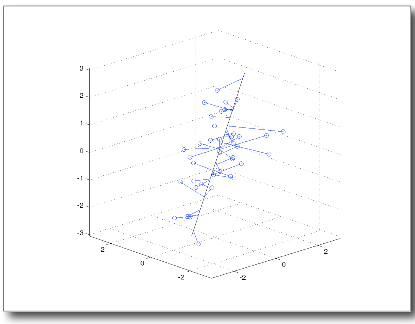
$P(x_1, x_2, \ldots, x_n)$

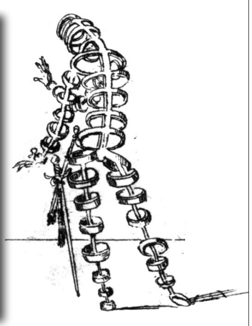
$Q(y_1, y_2, \ldots, y_n)$

$$[d(P, Q)]^2 = \sum_{i=1}^{n} (x_i - y_i)^2.$$

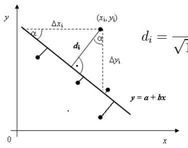
"The Flight of the Bumblebee"

Fitting 3D data by a straight line in 3D

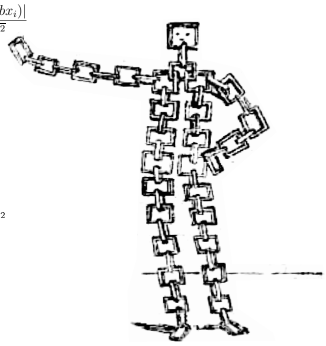




Orthogonal Linear Regression



$$d_i = \frac{\Delta y_i}{\sqrt{1+\tan^2\alpha}} = \frac{|y_i - (a+bx_i)|}{\sqrt{1+b^2}}$$



Following Legendre

$$E_\perp^2 = \sum_{i=1}^{n} \frac{[y_i - (a+bx_i)]^2}{1+b^2} = \frac{1}{1+b^2}\sum_{i=1}^{n}\Big[y_i - (a+bx_i)\Big]^2$$

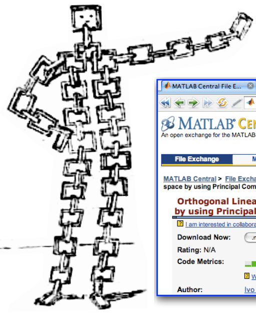
$$\frac{\partial E_\perp^2}{\partial a} = 0, \qquad \frac{\partial E_\perp^2}{\partial b} = 0,$$

we have two solutions:

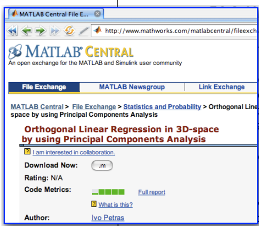
$$y = a_1 + b_1 x, \qquad y = a_2 + b_2 x \qquad b_1 b_2 = -1$$

Take that b that gives the smaller criterion value.

A MATLAB routine



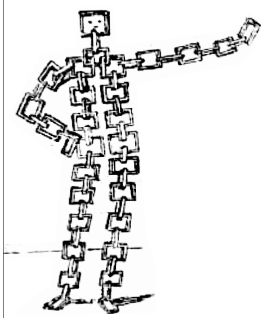
function [Err, N, P] =
fit_3D_data (XData, YData, ZData, ...etry, visualization, sod)



...lock -- x: axis
...lock -- y: axis
...lock -- z: axis
...approximation ('line','plane')
... ('on','off') -- default is 'on'
...nal distances ('on','off') -- default is 'on'

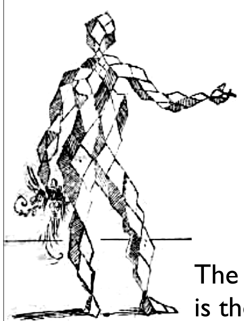
...ximation - sum of orthogonal distances
...r plane, direction vector for line
- P: point on plane or line in 3D space

Available at MATLAB Central File Exchange: http://www.mathworks.com/matlabcentral/fileexchange/



Let's borrow an idea from the control theory

State space for economies



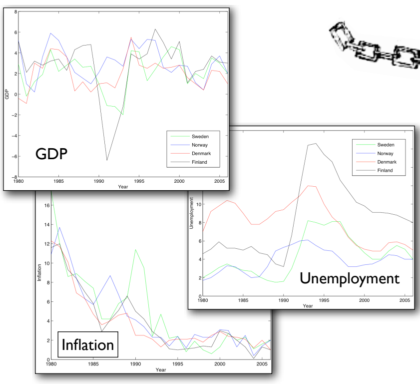
State variables $x(t), y(t), z(t)$:
- GDP rate
- unemployment rate
- inflation rate

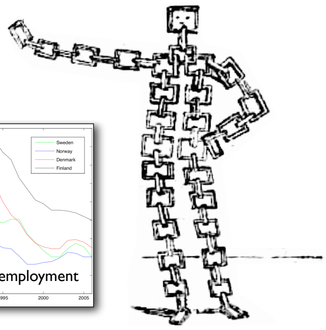
State of economy at $t = t_*$
is described by $\{x(t_*), y(t_*), z(t_*)\}$

The set of points $\{x(t), y(t), z(t)\},\ t \in [T_1, T_2]$
is the trajectory of the economy.

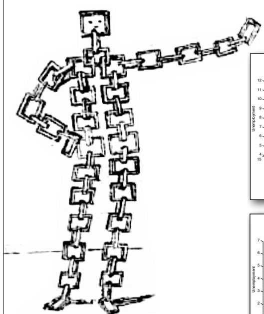
Scandinavian countries

Classical representation of the data:

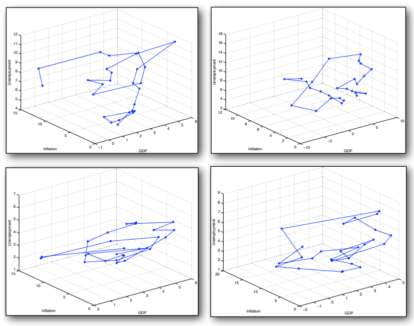




Scandinavian countries

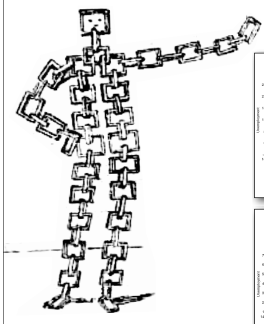


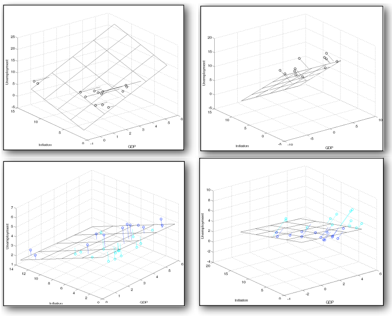
Trajectories of the economies:



Scandinavian countries

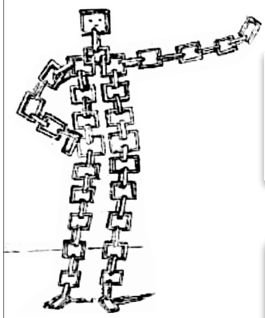
Characteristic planes
of the economies:

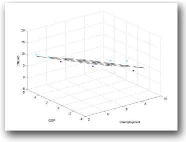




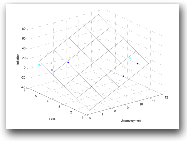
V_4 countries

Characteristic planes
of the economies:

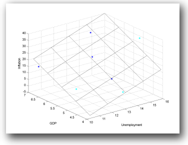




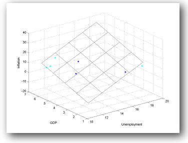
(a) Czech Republic



(b) Hungary

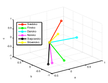


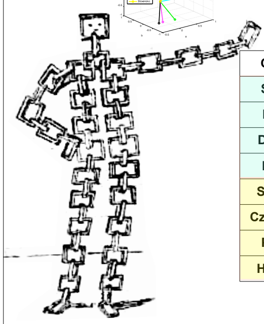
(c) Poland



(d) Slovak Republic

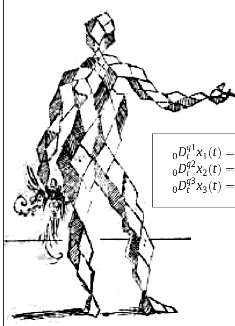
"Planes of national economies"





Country	Centroid	Normal vector	L_1 norm	L_∞ norm
Sweden	[2.24, 4.78, 4.56]	[0.56, 0.36, 0.73]	1.67	0.73
Finland	[2.60, 3.89, 8.37]	[-0.02, -0.80, -0.59]	1.42	0.80
Denmark	[1.95, 3.72, 7.73]	[0.94, 0.20, -0.26]	1.41	0.94
Norway	[3.02, 4.40, 3.76]	[-0.06, -0.25, -0.96]	1.29	0.96
Slovakia*	[13.87,4.55, 9.14]	[0.67, 0.71, -0.18]	1.57	0.71
Czech Rep*	[5.77, 1.84, 7.61]	[0.76, 0.45, 0.46]	1.67	0.76
Poland*	[−0.4,−0.9, 0.11]	[-0.40, -0.90, 0.11]	1.42	0.90
Hungary*	[0.73, 0.67,−0.05]	[0.73, 0.67, -0.05]	1.46	0.73

FDEs + Least Circles



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ELSEVIER

Modeling of the national economies in state-space: A fractional calculus approach

Tomáš Škovránek, Igor Podlubny, Ivo Petráš *

$$
\begin{aligned}
{}_0D_t^{q1}x_1(t) &= a_{11}x_1(t) + a_{12}x_2(t) + a_{13}x_3(t) + c_1,\\
{}_0D_t^{q2}x_2(t) &= a_{21}x_1(t) + a_{22}x_2(t) + a_{23}x_3(t) + c_2,\\
{}_0D_t^{q3}x_3(t) &= a_{31}x_1(t) + a_{32}x_2(t) + a_{33}x_3(t) + c_3.
\end{aligned}
$$

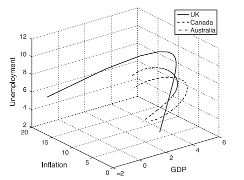
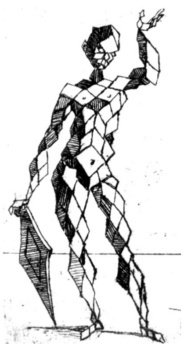


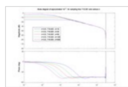
Fig. 4. State-space trajectories of national economies of chosen Commonwealth countries using the system in Eq. (10) for time period of years 1980–2009.

Conclusions



- A new approach to description of national economies has been presented.
- Gross domestic product, inflation, and unemployment rates were taken as state variables.
- The trajectory of the economy of each of the considered countries lies approximately in one plane.
- Economic development of each country can be associated with a corresponding plane in the state space (characteristic plane).
- Obtaining trajectories of national economies allows it classification as stable, unstable, or cyclic dynamical systems.

Software: Differintegration

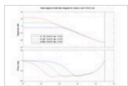


updated 10 months ago

Digital fractional-order differentiator and integrator - new IIR type by Ivo Petras

A new IIR type of the fractional-order differentiator and integrator. (tustin rule, digital differentiato…, digital integrator)

fx sysdfod=dfod3(n,T,r)

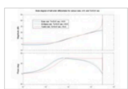


updated 10 months ago

Digital Fractional Order Differentiator/integrator - FIR type. by Ivo Petras

General FIR digital differentiator/integrator. (filter design, filter analysis, fractional calculus)

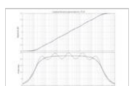
fx sysdfod=dfod2(n,T,r)



Digital Fractional Order Differentiator/integrator - IIR type by Ivo Petras

General IIR digital differentiator/integrator. (filter design, filter analysis, fractional calculus)

fx sysdfod=dfod1(n,T,a,r)



updated 9 years ago

Oustaloup-Recursive-Approximation for Fractional Order Differentiators by YangQuan Chen

Oustaloup-Recursive-Approximation for fractional order differentiator. (filter design, filter analysis, fractional order diff…)

fx [sys_foc]=ora_foc(r,N,w_L,w_H)

ora_foc_demo.m

Software: Mittag-Leffler related

Mittag-Leffler function by Igor Podlubny

Calculates the Mittag-Leffler function with desired accuracy. (mittagleffler functio…, fractional calculus, special functions)

fx [e]=mlf(alf,bet,c,fi)

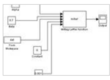
updated 21 days ago

Generalized Generalized Mittag-Leffler function by YangQuan Chen

Generalized Generalized Mittag-Leffler function in four parameters (mittagleffler functio…, generalized mittaglef…, math)

fx f=ggml_fun(a,b,c,d,x,eps0)

updated 4 years ago



updated 4 years ago

Mittag-Leffler function, M-file, cmex DLL, and S-function by Shayok Mukhopadhyay

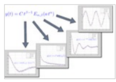
Mittag-Leffler function, M-file, cmex DLL, and S-function (mittag leffler, mfile, fractional calculus)

example

fx f=ml_fun(a,b,x,n,eps0)

param.m

Software: Mittag-Leffler related



updated 3 months ago

Fitting data using the Mittag-Leffler function by Igor Podlubny

Fitting data using the Mittag-Leffler function. (mittagleffler functio…, data fitting, fractional calculus)

fx mlffit1(XNodes, YDataPoints, mlfcoeffs0, Precision)

mlffit1demo.m

fx mlffit2(XNodes, YDataPoints, mlfcoeffs0, Precision)

Mittag-Leffler random number generator by Guido Germano

Mittag-Leffler pseudo-random number generator (statistics, mittagleffler distrib…, probability)

fx mlrnd(beta, gamma_t, m, n)

updated 4 years ago

Software: Matrix approach

Matrix approach to distributed-order ODEs and PDEs by Igor Podlubny

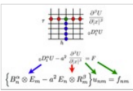
Basic functions for using matrix approach for distributed-order differential equations, and demos. (differential, discretization, distributedorder syst…)

dobagleytorvik.m

fx doban(phi, alphaFromTo, alphaStep, tN, tStep)

dodiffusionwave.m

updated 5 months ago



updated 3 years ago

Matrix approach to discretization of ODEs and PDEs of arbitrary real order by Igor Podlubny

Functions illustrating matrix approach to discretization of ODEs / PDEs with fractional derivatives. (differential, discretization, fractional differenti…)

fx ranort(alpha,N,h)

Matrix approach to discretization of ODEs and PDEs of arbit…

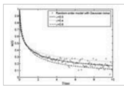
fx eliminator(n, ROWS)

Software: Other tools for FDEs

Predictor-corrector PECE method for fractional differential equations by Roberto Garrappa

Solves initial value problems for fractional differential equations (mathematics, fractional differenti…, predictor corrector)

fx fde12(alpha,fdefun,t0,tfinal,y0,h,param,mu,mu_tol)



updated almost 2 years ago

Predictor-Corrector Method for constant, variable and random fractional order relaxation equation by Hongguang Sun

Predictor-Corrector Method for fractional VO and RO equation (self_rating, fractional calculus v…)

fx sunpredictorcorrector

fx p=fractPC(a,b);

Solution of Fractional Optimal Control Problems by Christophe Tricaud

Solution of Fractional Optimal Control Problems using Rational Approximation (control design)

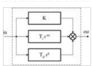
fx sys_Dh(neq,t,x,u)

fx sys_Dg(neq,t,x0,xf)

fx sys_activate

updated 3 years ago

Software: Fractional-order control



updated 11 months ago

Discrete Fractional-Order PID Controller by Ivo Petras

Provides a transfer function of the fractional-order PID controller for given parameters. (pid, fractional controller, noninteger order cont…)

fx DFOC(K, Ti, Td, m, d, Ts, n, method)

fx requireFEXpackage(FEXSubmissionID)

Impulse response invariant discretization of fractional order low-pass filters by YangQuan Chen

Discretize [1/(\tau s +1)]^r with "r" a real number (filter design, filter analysis, fractional calculus)

fx [sr]=irid_folpf(tau,r,Ts,norder)

updated 4 years ago

Step response invariant discretization of fractional order integrators/differentiators by YangQuan Chen

Compute a discrete-time finite dimensional (z) transfer function to approximate s^r, r = real number (filter design, filter analysis, fractional calculus)

fx [sr]=srid_fod(r,Ts,norder)

updated 4 years ago

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updated 4 years ago

ninteger by Duarte Valério

Toolbox to help developing fractional order controllers and assess their performance. (fractional, fractional plants, fractional control)

fx bodeFr.m

fx contfrac (x, n)

fx contfraceval (a, b, n)

updated 4 years ago

Software: Fractional-order control

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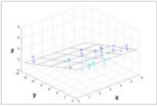
updated 4 years ago

Software: Least Circles related

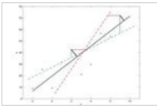


updated 11 months ago

Orthogonal Linear Regression in 3D-space by using Principal Components Analysis by Ivo Petras

Orthogonal Linear Regression by using PCA (pca, orthogonal regression, total least squares)

fx fit_3D_data(XData, YData, ZData, geometry, visualization, s...



updated 12 months ago

Total Least Squares Method by Ivo Petras

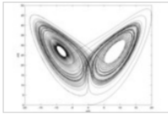
Mathematical method known as total least squares or orthogonal regression or error-in-variables. (total least squares, svd, errorinvariables)

fx corrindex(XData, YData, YDataM, par_number)

demo3Dplane.m

demoIdent.m

Software: Fractional-order chaos



updated 10 months ago

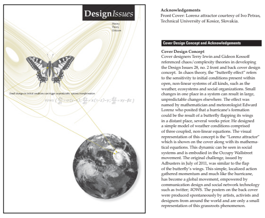
Fractional Order Chaotic Systems by Ivo Petras

Numerical solutions of the fractional order chaotic systems. (numerical solution, attractor, chaos)

Demo_FOChS.m

fx [T, Y]=FO3CNN(parameters, orders, TSim, Y0)

fx [T, Y]=FOArneodo(parameters, orders, TSim, Y0)



http://www.mitpressjournals.org/toc/desi/28/2

Homework

Why did I arrange a picture of us
at this particular place in Ravello?

Homework

Why did I arrange a picture of us
at this particular place in Ravello?

This should help you

Thank you!

Lecture slides, videos, lecture notes,
and other related material and links:

http://people.tuke.sk/igor.podlubny/ravello2012/